On the Concept of Quantum State Reduction: Inconsistency of the Orthodox View

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Abstract

The argument is re-examined that the program of deriving the rule of state reduction from the Schrödinger equation holding for the object-apparatus composite system falls into a vicious circle or an infinite regress called the von Neumann chain. It is shown that this argument suffers from a serious physical inconsistency concerning the causality between the reading of the outcome and the state reduction. A consistent argument which accomplishes the above program without falling into the circular argument is presented.

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I. INTRODUCTION

Quantum mechanics includes a dualism concerning the principle of state change. The Schrödinger equation, on the one hand, governs the state change caused by time evolution. The rule of state reduction, on the other hand, governs the state change caused by measurement. The dualism is justified as long as the state change of only one system is concerned. The Schrödinger equation holds true only when the system is isolated, but every measurement accompanies the interaction with the measuring apparatus so that the rule of state reduction holds true only when the system is not isolated. The dualism is therefore justified [1, p. 420].

Accepting that every measurement accompanies the interaction between the object and the apparatus at all, one can expect that the rule of state reduction can be derived from the Schrödinger equation holding for the composite system of the object and the apparatus during the measurement. A negative view, however, prevails against this program. According to that view, the Schrödinger equation for the composite system transforms the problem of a measurement on the object to the problem of an observation on the apparatus, but in order to derive the rule of state reduction holding for the object one still needs the rule of state reduction applied to the composite system [2, p. 329]. This implies that the program of deriving the rule of state reduction from the Schrödinger equation holding for the object-apparatus composite system falls into a vicious circle sometimes called von Neumann's chain [3, Section 11.2].

The purpose of this paper is to show that the above argument, usually called the orthodox view of measurement theory, includes a serious physical inconsistency and then to present a consistent argument which derives the rule of state reduction from the Schrödinger equation of the composite system without falling into the vicious circle.

In this paper, we are confined to the state reduction caused by a measurement of an observable with nondegenerate purely discrete spectrum satisfying the repeatability hypothesis. Sections II–VII review with elaboration the most basic part of measurement theory originated with von Neumann [1]. Section II presents postulates for quantum mechanics and defines state reduction. Section III introduces the notion of nonselective measurement and shows that a nonselective measurement causes a state change in quantum mechanics whereas it is not the case in classical mechanics. Section IV concludes the existence of an interaction between the measured object and the apparatus in every measurement. Section V shows that the rule of state reduction is equivalent to the repeatability hypothesis. Section VI introduces the projection postulate as the rule of state reduction in the case where the observable has degenerate spectrum. Section VII derives a necessary condition for a unitary operator to represent the measuring interaction. The condition determines the form of the unitary operator representing the measuring interaction leading to state reduction. The problem is then formulated as whether the unitary operator of this form is sufficient for deriving the rule of state reduction. Section VIII reviews the orthodox view along with Wigner's argument [2] that claims that the unitary operator does not lead to the rule of state reduction without appealing to the rule of state reduction, the projection postulate, for the measurement of the pointer position. Section IX shows that the orthodox view suffers from a serious physical inconsistency concerning the causality between the reading of the outcome and the state reduction. Section X presents a consistent argument which derives the

rule of state reduction from the Schrödinger equation holding for the object-apparatus composite system without appealing to the projection postulate for the pointer-measurement. Concluding remarks are provided in Section XI.

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle \phi_n|\psi \rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

III. ORIGIN OF MEASUREMENT THEORY

The state reduction can be thought to be caused by the following two factors: the dynamical change of the system and the change of the observer's knowledge. In order to separate these factors, let us suppose that we terminate the procedure of measurement of the observable A just before the observer's reading of the outcome; this procedure is called a *nonselective measurement*. It follows from postulates (A1) and (A2) that the nonselective measurement leaves the system in the mixture of the states $|\phi_n\rangle\langle\phi_n|$ with the probability $|\langle\phi_n|\psi\rangle|^2$ and therefore yields the following state change:

$$|\psi\rangle\langle\psi| \mapsto \sum_n |\langle\phi_n|\psi\rangle|^2 \, |\phi_n\rangle\langle\phi_n|. \tag{3.1}$$

Since a nonselective measurement does not change the observer's knowledge, this state change is considered to be caused entirely by the dynamical factor.

Even in classical mechanics, when the outcome of measurement is obtained, the information on the outcome changes the observer's knowledge and the probabilistic description of the state of the system changes according to the Bayes principle, which is formulated as follows: Let X, Y be two (discrete) random variables. Suppose that we know the joint probability distribution $\Pr\{X = x, Y = y\}$. Then, the prior distribution of X is defined as the marginal distribution of X, i.e.,

$$\Pr\{X = x\} = \sum_y \Pr\{X = x, Y = y\}. \tag{3.2}$$

If one measures Y, the *information* "$Y = y$" changes the probability distribution of X for any outcome y. The posterior distribution of X is defined as the conditional probability distribution of X given $Y = y$, i.e.,

$$\Pr\{X = x | Y = y\} = \frac{\Pr\{X = x, Y = y\}}{\sum_x \Pr\{X = x, Y = y\}}. \tag{3.3}$$

This principle of changing the probability distribution from the prior distribution to the posterior distribution is called the *Bayes principle*.

Nonetheless, a nonselective measurement in classical mechanics causes no change in the system. Therefore, it is a characteristic feature of quantum mechanics that a nonselective measurement changes the system dynamically, and it is the origin of von Neumann's measurement theory to explain this change.

IV. EXISTENCE OF MEASURING INTERACTION

In quantum mechanics the state of an isolated system changes dynamically according to the Schrödinger equation, but this state change does not change the entropy of the system. On the other hand, the state change (3.1) caused by the nonselective measurement increases the entropy of the measured system, and hence this process of state change cannot be described by the Schrödinger equation of the measured system [1, p. 388]. It follows that this dynamical change of state must be caused by the interaction between the measured object and the measuring apparatus, a system external to the measured object including every system interacting with the measured object. Thus, from the basic postulates of quantum mechanics we have derived the existence of measuring interaction, which is neglected in classical mechanics. Since our discussion concerns only nonselective measurements, we do not need to mention the function of consciousness, although the conventional argument mentions the psycho-physical parallelism [1, pp. 418–420].

V. REPEATABILITY HYPOTHESIS AND STATE REDUCTION

In the measurement axiom (A2), state reduction is described as a change of the state of the object. In order to consider state reduction together with the interaction between the object and the apparatus, it is desirable to describe it independently of particular descriptions of states of systems. As one of such descriptions, von Neumann showed that (A2) is equivalent to the following *repeatability hypothesis* [1, pp. 213–218, 335]:

(M) *If a physical quantity is measured twice in succession in a system, then we get the same value each time.*

In fact, according to (M) the state of the object just after the first measurement is the eigenstate corresponding to the outcome, and in the nondegenerate case it is determined uniquely so that we have (A2). It is obvious that (M) follows from (A2).

VI. PROJECTION POSTULATE

In this paper, we are devoted to measurements of observables with nondegenerate discrete spectrum. In the conventional discussion explaining state reduction, however, we need to consider a measurement on the object-apparatus composite system and we need the statistical formula and the measurement axiom for observables with degenerate spectrum. The statistical formula for the discrete observable $X = \sum_n a_n E_n$ to be measured in the state (density operator) ρ is given as follows:

(B1) *The eigenvalue a_n is obtained as the outcome with the probability* $\mathrm{Tr}[E_n \rho]$.

In the degenerate case, the eigenstate corresponding to an eigenvalue is not determined uniquely, and hence (A2) is ambiguous. Moreover, the repeatability hypothesis (M) implies that the object is left in an eigenstate corresponding to the outcome, but (M) does not determine the eigenstate in which the object is left. For determining this eigenstate, von Neumann posed no special principle [1, p. 348], but later Lüders proposed a principle which has been widely accepted [4]. According to his principle, if the measurement of the discrete observable X with spectral decomposition $X = \sum_n a_n E_n$ is carried out in the state (density operator) ρ and leads to the outcome a_n, then:

(B2) *At the time just after measurement, the object is left in the state* $E_n \rho E_n / \mathrm{Tr}[E_n \rho]$.

This principle is called the *projection postulate*, because the eigenstate ϕ_n provoked by the measurement is chosen by the projection E_n onto the corresponding eigenspace so that $\phi_n = E_n \psi / \|E_n \psi\|$ for the initial state ψ. It is clear that statements (B1) and (B2) imply (A1) and (A2) as special cases. It follows from (B1) and (B2) that the nonselective measurement of X leads to the state change such as

$$\rho \mapsto \sum_n E_n \rho E_n. \tag{6.1}$$

VII. INTERACTION BETWEEN THE OBJECT AND THE APPARATUS

We shall turn to the discussion on the measurement of the discrete observable $A = \sum_n a_n |\phi_n\rangle\langle\phi_n|$ with nondegenerate eigenvalues. In section IV, we have concluded that the state change (1) in the nonselective measurement must be caused by the interaction between the object and the apparatus. Then, what is this interaction?

Let us suppose that the object is in the eigenstate ϕ_n of the observable A pertaining to the eigenvalue a_n. By the statistical formula, the outcome is a_n with probability one. Hence, the measurement changes the position of the pointer in the apparatus from the original position to the position a_n on the scale.

Let B be the observable corresponding to the position of the pointer in the apparatus, and ξ the original state of the apparatus. Generally it is only required [1, p. 439] that the eigenvalues $\{b_n\}$ of B are in one-to-one correspondence with the eigenvalues $\{a_n\}$ of A, but we assume for simplicity that the observable B has also the same eigenvalues, $a_1, a_2, \ldots$, as A. In the Hilbert space of the composite system of the object and the apparatus, the observables A and B are represented by the self-adjoint operators $A \otimes 1$ and $1 \otimes B$ respectively.

The state change due to the interaction is represented by a unitary operator U on the Hilbert space of the composite system:

$$U : \phi_n \otimes \xi \mapsto U(\phi_n \otimes \xi). \tag{7.1}$$

According to the statistical formula (A1), the state $U(\phi_n \otimes \xi)$ after the interaction must be the eigenstate of $1 \otimes B$ pertaining to the eigenvalue a_n. In fact, the position of the pointer takes the value a_n with probability one after the interaction, and this implies that the state $U(\phi_n \otimes \xi)$ is the eigenvector of $1 \otimes B$ pertaining to the eigenvalue a_n.

On the other hand, according to the repeatability hypothesis (M) the state $U(\phi_n \otimes \xi)$ is the eigenvector of $A \otimes 1$ pertaining to the eigenvalue a_n. In fact, if the observer were to measure A in this state again, then the observable A would be measured twice in succession and hence the outcome would be a_n with probability one. This implies that the state $U(\phi_n \otimes \xi)$ is the eigenstate of $A \otimes 1$ pertaining to the eigenvalue a_n.

Suppose here that the eigenvalues of B are also nondegenerate. Then the state that satisfies the above two conditions is represented by the state vector $\phi_n \otimes \xi_n$, where ξ_n is an arbitrary eigenvector of B with unit length pertaining to the eigenvalue a_n. In order to represent the measurement the unitary U must satisfy the following relation

$$U(\phi_n \otimes \xi) = \phi_n \otimes \xi_n. \tag{7.2}$$

If the unitary operator U satisfies the above condition, then for the arbitrarily given original state $\psi = \sum_n c_n \phi_n$ we have by linearity

$$U(\psi \otimes \xi) = \sum_n c_n \phi_n \otimes \xi_n. \tag{7.3}$$

Thus the problem is whether equation (7.3) is a sufficient condition for the unitary operator U to represent the measuring interaction or whether, in other words, equation (7.3) implies (A1) and (A2) even when ψ is a superposition of the eigenstates ϕ_n. If equation (7.3) were not a sufficient condition, further interaction — though ineffective in the case where the object is initially in the eigenstate — might be necessary for the explanation leading to the state reduction.

VIII. THE ORTHODOX VIEW

The conventional approach adopted by most of the text books on measurement theory, the so-called orthodox view, is negative about the above problem. The orthodox view holds that (7.3) does not imply the measurement axiom (A2). The argument runs as follows.

The state transformation by the unitary U,

$$\left(\sum_n c_n \phi_n\right) \otimes \xi \mapsto \sum_n c_n \phi_n \otimes \xi_n, \tag{8.1}$$

makes a one-to-one correspondence between the state of the object and the state of the apparatus. The state of the object is mirrored by the state of the apparatus, and the problem of a measurement on the object is transformed into the problem of an observation on the apparatus [2]. If the observer observes the pointer position of the apparatus to obtain the outcome of measurement, the state in the right-hand side of (8.1) changes as follows:

$$|\sum_n c_n \phi_n \otimes \xi_n\rangle\langle\sum_n c_n \phi_n \otimes \xi_n|$$
$$\mapsto \sum_n |c_n|^2 |\phi_n \otimes \xi_n\rangle\langle\phi_n \otimes \xi_n|. \tag{8.2}$$

The state change in (8.2) is derived by the projection postulate (B2) applied to the state change caused by the measurement of the observable $1 \otimes B$ of the composite system. According to the projection postulate (B2), the new state is the mixture of the states $\phi_n \otimes \xi_n$, and hence when the outcome a_n is obtained, a system in the state $\phi_n \otimes \xi_n$ is selected from the ensemble described by the right-hand side of (8.2):

$$\sum_n |c_n|^2 |\phi_n \otimes \xi_n\rangle\langle\phi_n \otimes \xi_n| \mapsto |\phi_n \otimes \xi_n\rangle\langle\phi_n \otimes \xi_n|. \tag{8.3}$$

Finally, the composite system is in the state $\phi_n \otimes \xi_n$, and this implies that the object is led to the state ϕ_n.

Nevertheless, if we describe further the measuring process which leads to the state change (8.2) in terms of the coupling with the second apparatus, having an orthonormal system $\{\xi'_n\}$ and being prepared in a state ξ', measuring the pointer position in the first apparatus, then instead of (8.2) we have the following state change:

$$(\sum_n c_n \phi_n \otimes \xi_n) \otimes \xi' \mapsto \sum_n c_n \phi_n \otimes \xi_n \otimes \xi'_n. \tag{8.4}$$

From this state change, we cannot conclude that the measurement leads the object with the outcome a_n to the state ϕ_n. The original problem of explaining the state reduction caused by the first apparatus is not solved but only transferred to the the problem of explaining the state reduction caused by the second apparatus [3, Section 11.2]. This vicious circle is often called *von Neumann's chain*.

IX. INCONSISTENCY OF THE ORTHODOX VIEW

A difficulty in the orthodox view is to apply the projection postulate to the object-apparatus composite system in order to show that the state of the object that leads to the outcome a_n is in the state ϕ_n at the time just after measurement. This argument suffers from the circular argument that assumes the rule of state reduction for the composite system in order to explain the rule of state reduction for the object. The conventional studies of measurement theory, however, have not detected any physical inconsistency or empirical inadequacy of the above argument in the orthodox view and have aimed at circumventing the above circular argument, an epistemological difficulty, by adding, for instance, the element of macroscopic nature of the measuring apparatus [5,6].

In fact, the above argument leading to the state reduction has been generalized to the following argument for any measurements to determine the state change caused by measurement conditional upon the outcome: Let us given the initial state ρ of the object, the initial state of the apparatus σ, and the unitary evolution operator U of the object-plus-apparatus. Then, compute the state of the composite system just after the interaction as $U(\rho \otimes \sigma)U^\dagger$ and apply the projection postulate to the measurement of the pointer observable B in the

apparatus, and the state ρ_{a_n} just after the measurement conditional upon the outcome a_n is given by

$$\rho_{a_n} = \frac{\mathrm{Tr}_{\mathbf{A}}[(I \otimes E^B(a_n))U(\rho \otimes \sigma)U^\dagger(I \otimes E^B(a_n))]}{\mathrm{Tr}[(I \otimes E^B(a_n))U(\rho \otimes \sigma)U^\dagger(I \otimes E^B(a_n))]}, \tag{9.1}$$

where $E^B(a_n)$ is the projection operator onto the eigenspace of B corresponding to the eigenvalue a_n and $\mathrm{Tr}_{\mathbf{A}}$ stands for the partial trace over the Hilbert space of the apparatus. This unitary-evolution-plus-projection-postulate argument has been a standard argument for determining the general state reduction, see for example [7–11].

The purpose of this section is, despite the conventional arguments, to show that the orthodox view suffers from a serious physical inconsistency concerning the causality between the reading of the outcome and the state reduction.

In order to explain the rule of state reduction in terms of the time evolution of the object-plus-apparatus, it is necessary to clarify the meanings of the words the "time of measurement" and the "time just after measurement" in the context as to what happens in the object and the apparatus. Let us suppose that one measures an observable $A = \sum_n a_n |\phi_n\rangle\langle\phi_n|$ of the object in the state ψ at the time t. The measurement, carried out by an interaction with the apparatus, takes finite time $\Delta t > 0$. Thus, the object interacts with the apparatus from the time t to $t + \Delta t$ and is free from the apparatus after the time $t + \Delta t$. It follows that the time of measurement is the time t, the time just after measurement is $t + \Delta t$, and that the object is in the state ψ at the time of measurement. The statistical formula (A1) means, in this case, that the observer obtains the outcome a_n with probability $|\langle\phi_n|\psi\rangle|^2$. The measurement axiom (A2) means that the object that leads to the outcome a_n is in the state ϕ_n at the time $t + \Delta t$. Moreover, the repeatability hypothesis (M) means that if the observable A is measured at the time $t_1 = t + \Delta t$ again in the same object then the outcome coincides with the one obtained by the measurement of A at the time t.

In the orthodox view, the state changes given by (8.1) and (8.2) represent dynamical changes of the system, and the state change (8.3) represents a change of the knowledge of the observer. The state change (8.1) represents the interaction between the object and the apparatus. The state change (8.2) represents the interaction between the "apparatus" and the "apparatus measuring the apparatus". It follows that the state change (8.1) shows that the object-plus-apparatus is in the state $(\sum_n c_n \phi_n) \otimes \xi$ at the time t and in the state $\sum_n c_n \phi_n \otimes \xi_n$ at the time $t + \Delta t$.

Suppose that the state change (8.2) takes time τ. Then, it is at the time $t + \Delta t + \tau$ that the object-plus-apparatus turns to be in the state described by the right-hand side of (8.2). Since the state change (8.3) represents the change of the observer's knowledge, it does not accompany the change of time so that at the time $t + \Delta t + \tau$ the object turns to be in the state ϕ_n as the result of the state reduction. In other words, the orthodox view leads to the conclusion that the state reduction occurs at the time $t + \Delta t + \tau$ which is later in time τ than the time $t + \Delta t$ just after measurement. Thus, if τ is not negligible in the time scale of the time evolution of the object then this contradicts the measurement axiom that the state reduction leaves the system in the state ϕ_n at the time $t + \Delta t$.

Since the object is free from the apparatus after the time $t_1 = t + \Delta t$, one can make the object interact with the second apparatus at the time t_1. If this apparatus also measures A, according to the repeatability hypothesis it is predicted, and will be confirmed by experiments, that the outcome from the first apparatus and the outcome from the second are

always the same. But, this fact cannot be explained by the orthodox view which concludes that the state reduction occurs at the time $t_1 + \tau$.

Is τ negligible in the time scale of the time evolution of the object? In general, the process of the state change (8.2) is regarded as a process in which a macroscopic instrument operates or a directly-sensible variable feature is produced — otherwise, the state change in the apparatus measurement might not necessarily satisfy the repeatability hypothesis — and hence the duration τ of this process cannot be negligible in the time scale of the time evolution of the microscopic measured object.

Consider, for instance, the experiment in which the light is scattered by an atom in a low intensity atom beam. Regarding the paths before the collision as known, the measurement of the path of the photon after the collision suffices to determine the point of scattering. In order to measure the position of the atom (at the time of collision) twice in succession in this method, suppose to use two nearly placed light beams I and II; see FIG. 1. Suppose that the atom interacts with the beam I from the time t to t' and with the beam II from t_1 to t_1' and that $t_1 - t'$ is so small that the time evolution of the atom in this period can be neglected — hence, we can put $t_1 = t'$. Suppose that the photon scattered from the beam I is detected by a photoelectric detector at the time t'', and the one from the beam II is detected by another photoelectric detector at the time t_1''. In this experiment, the collision is accomplished in quite a short time and the photoelectric detectors are necessary to place sufficiently far from the light beams, so that it is taken for granted in scattering theory [12, p. 375] that

$$t' - t \ll t'' - t'. \tag{9.2}$$

It is taken for granted from the Compton-Simons experiment that there is the uncertainty of the position at which the beam I is scattered depending on the initial state of the atom but that the position of the scattering from the beam II is always near the position of the scattering from the beam I. It follows that this experiment can be considered as the position measurement of the atom satisfying the repeatability hypothesis [1, pp. 212–214]. In this example, the state change (8.1) corresponds to the interaction between the atom and the light beam, and hence we have $\Delta t = t' - t$. On the other hand, the apparatus corresponds to the scattered photon, the state change (8.2) corresponds to the process including the free propagation of the photon after scattering and the interaction between the photon and the photoelectric detector, and hence we have $\tau = t'' - t'$. Thus, from (9.2) we have

$$\Delta t \ll \tau, \tag{9.3}$$

and consequently we cannot neglect τ. This means that the orthodox view claims that the state reduction of the atom occurs after the photon is detected despite the fact that the state reduction of the atom occurs just after the scattering of the light.

The inconsistency of the orthodox view is in the claim of causality between the reading of the outcome and the state reduction such that the state change (8.2) of the composite system causes the state reduction of the object system. It is obvious, however, that such causality does not exists, since the result, the state reduction, occurs before the cause, the reading of the outcome or the manifestation of the directly-sensible variable feature. It is not the case that the observer's knowing or reading of the outcome at the time $t'' = t + \Delta t + \tau$ causes the state reduction of the object at the time $t' = t + \Delta t$. But, it is the case that

by knowing or reading of the outcome at the time t'' the observer obtains the information to determine the state of the object at the time t'. The orthodox view confuses the time at which the outcome of measurement is obtained and the time at which the object is left in the state determined by the outcome. Or, in other words, it confuses the time just after the reading of the outcome and the time just after the interaction between the object and the apparatus. There is no causality relation between the outcome and the state just after measurement but there is coincidence between them yielded by the measuring interaction.

X. NEW INTERPRETATION

Our new interpretation presented in the following does not includes the inconsistency of the orthodox view discussed in the previous section. Moreover, the state reduction can be explained only from (8.1) without assuming the process (8.2) so that the circular argument of the von Neumann chain is circumvented.

As in the preceding section, suppose that the observer measures the observable $A = \sum_n a_n |\phi_n\rangle \langle \phi_n|$ of the object in the state ψ at the time t. The object interacts with the apparatus from the time t to $t + \Delta t$ and is free from the apparatus after the time $t + \Delta t$. The repeatability hypothesis (M) means that if the observer measures A at the time $t_1 = t + \Delta t$ again then the outcome coincides with the outcome of the measurement at the time t. As shown previously, the measurement axiom (A2) is equivalent to the repeatability hypothesis (M). Hence, if the statistical formula (A1) and the repeatability hypothesis (M) is derived from (7.3), it is demonstrated that the state reduction is derived from (7.3). Let $\Pr\{B(t+\Delta t) = a_n\}$ be the probability of obtaining the outcome a_n when the pointer position $B = \sum_n a_n |\xi_n\rangle \langle \xi_n|$ is measured at the time $t + \Delta t$. By (7.3) and the statistical formula (B1) for the degenerate observable $1 \otimes B$ we have

$$
\begin{aligned}
&\Pr\{B(t + \Delta t) = a_n\} \\
&= \langle \sum_k c_k \phi_k \otimes \xi_k | (1 \otimes |\xi_n\rangle \langle \xi_n|) | \sum_k c_k \phi_k \otimes \xi_k \rangle \\
&= |c_n|^2.
\end{aligned}
$$

Let $\Pr\{A(t) = a_n\}$ be the probability that the measurement of A at the time t leads to the outcome a_n. Since this outcome is obtained as the outcome of the measurement of B at the time $t + \Delta t$, we have

$$
\Pr\{A(t) = a_n\} = \Pr\{B(t + \Delta t) = a_n\} = |c_n|^2. \tag{10.1}
$$

Thus if we regard this process as the measurement of A — namely, if we interpret the outcome of the measurement of B at the time $t + \Delta t$ as the outcome of the measurement of A at the time t — then it is shown that this measurement satisfies the the statistical formula (A1).

We shall show that this measurement satisfies the measurement axiom (A2) in the following. Since the measurement axiom (A2) is equivalent to the repeatability hypothesis (M), we need only to show that this measurement satisfies the repeatability hypothesis (M).

In order to show the last statement, it suffices to show that if the observer measures A again at the time $t_1 = t + \Delta t$, immediately after the first measurement, then the outcome

of the first measurement at the time t and that of the second at the time t_1 are always the same. Let $\Pr\{A(t) = a_n, A(t_1) = a_m\}$ be the joint probability that the first outcome is a_n and the second outcome is a_m.

The outcome of the first measurement of A at the time t is the same as the outcome of the measurement of the pointer position B at the time $t_1 = t + \Delta t$. Since the measurements of B and A at the time t_1 does not interferes each other, the joint probability distribution of the outcomes of these two measurements satisfies the statistical formula for the simultaneous measurements:

$$\Pr\{B(t_1) = a_n, A(t_1) = a_m\}$$
$$= |\langle \phi_m \otimes \xi_n| \sum_k c_k \phi_k \otimes \xi_k \rangle|^2$$
$$= \delta_{m,n}|c_n|^2.$$

Thus if $m \neq n$ then we have

$$\Pr\{A(t) = a_n, A(t_1) = a_m\}$$
$$= \Pr\{B(t_1) = a_n, A(t_1) = a_m\}$$
$$= 0.$$

It follows that the outcome of the first measurement and that of the second are always the same. Therefore, this process satisfies the repeatability hypothesis (M) and hence satisfies the measurement axiom (A2).

We have thus demonstrated that the unitary operator U satisfying (7.3) represents the interaction between the object and the apparatus and leads to the state reduction in the object at the time just after measurement.

XI. CONCLUDING REMARKS

It follows from the basic postulate requiring the state reduction that even in the case where the observer obtains no information from the measurement, namely the case of non-selective measurement, the state of the system changes. This change is not accompanied with the change of knowledge so that it is purely dynamical, but it is irreversible so that it cannot be described by the Schrödinger equation of the object. The only way to explain this by the rules of quantum mechanics is to derive this change from the Schrödinger equation of a larger system than the object, which describes the interaction between the object and the apparatus. This interaction is turned on during a finite time interval, from the time t of measurement to the time $t + \Delta t$ just after measurement. After the object-apparatus interaction, the object turns to be free from the apparatus again. Thus, the state reduction describes the state change from the time t to the time $t + \Delta t$ conditional upon the outcome of measurement.

The state change in the nonselective measurement is derived without any difficulties from the interaction between the object and the apparatus. In fact, the state change (3.1) is explained as the open system dynamics of the object yielded by the unitary evolution of the object-apparatus composite system described by the unitary U in (7.3), i.e.,

$$\sum_n |\langle \phi_n | \psi \rangle|^2 \, |\phi_n\rangle\langle\phi_n| = \mathrm{Tr}_{\mathbf{A}}[U|\psi \otimes \xi\rangle\langle\psi \otimes \xi|U^\dagger], \tag{11.1}$$

where $\mathrm{Tr}_{\mathbf{A}}$ is the partial trace over the Hilbert space of the apparatus [13, p. 136]. The problem is to explain the change of state dependent on the outcome, namely the state reduction. The answer of the orthodox view to this problem is that the state reduction of the object is resulted from the state reduction of the object-plus-apparatus caused by the measurement of the pointer position carried out after the object-apparatus interaction. Applying the above argument that state reduction needs the time for the nonselective measurement to the measurement of the pointer position, it is concluded that the state reduction, explained by the orthodox view, occurs apparently later than the time $t + \Delta t$ at which the state reduction should occur. This time difference leads to the detectable difference as to whether the outcomes obtained by measuring the same object twice in succession satisfy the repeatability hypothesis, and hence we can conclude that the inconsistency of the the orthodox view can be tested by an experiment.

The photon scattering experiment from the atom beam in an atom interferometer has been realized already by Chapman *et al.* [14]. The double scattering *gedanken* experiment suggested in Section IX will be realized in future along with a similar experimental setting with the additional second laser beam for the repeated measurement of the point of scattering of a single atom, if a conceivable difficulty can be circumvented in distinguishing the case where two detected photons from the two beams have been scattered by a single atom from the other cases.

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FIGURES

FIG. 1. Repeated position measurements by photon scattering from an atom.